Routing optimization on networks: a statistical physics approach

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Collaborations: Fabrizio Altarelli, Alfredo Braunstein, Luca Dall'Asta, David Saad, C. H. Yeung

Background

- Postdoc at Santa Fe Institute
- PhD at LPTMS, Paris Sud, Supervisors: S. Franz and S. N. Majumdar
- Master degree at University of Padova (F. Baldovin and E. Orlandini)
- Erasmus at Imperial College 2011 + summer internship at Centrica energy
- Bachelor degree at University of Padova

Summary

1. Motivation: routing problems on networks
2. Node disjoint path
3. Edge disjoint path
4. General Traffic optimization

- *Shortest node-disjoint paths on random graphs*, C. De Bacco, S. Franz, D. Saad, C.H. Yeung, J. Stat. Mech. (2014) P07009
- *The edge-disjoint path problem on random graphs by message-passing*, F. Altarelli, A. Braunstein, L. Dall'Asta, C. De Bacco and S. Franz, arxiv 1503.00540, 2015

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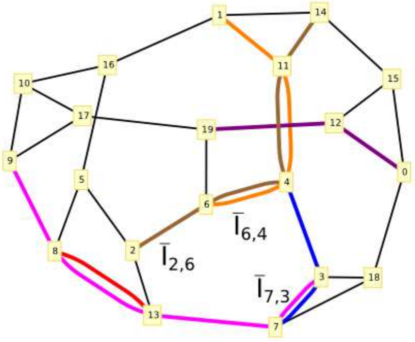
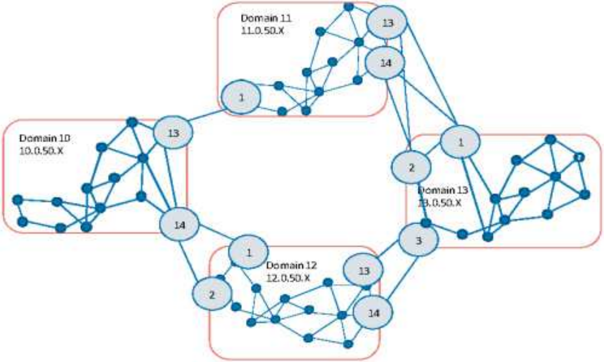
The routing optimization problem

- M communications. Ex. $M = 6$ (here denoted by colors: purple=1,red=2,brown=3,orange=4,blue=5, violet=6).
- $\bar{I}_{ij}$ denotes current passing along edge (ij). Ex.: $\bar{I}_{6,4} = (0, 0, -1, 1, 0, 0)$, $\bar{I}_{2,6} = (0, 0, -1, 0, 0, 0)$ and $\bar{I}_{7,3} = (1, 0, 0, 0, -1, 0)$
- A cost $f(||\bar{I}_{ij}||)$ can penalize path length and traffic



Goal: what is the optimal way to route communications?

Example: Routing and Wavelength assignment (RWA)



Constraints:

- One λ per communication
- Two communications sharing one link must be assigned two different λ's
- ... and the shorter the route the faster the communication

Main properties

- Common proposed algorithms: Integer Linear Programming, greedy, Ant Colony Optimization etc...
- Non-local interaction between paths $\rightarrow$ global optimization

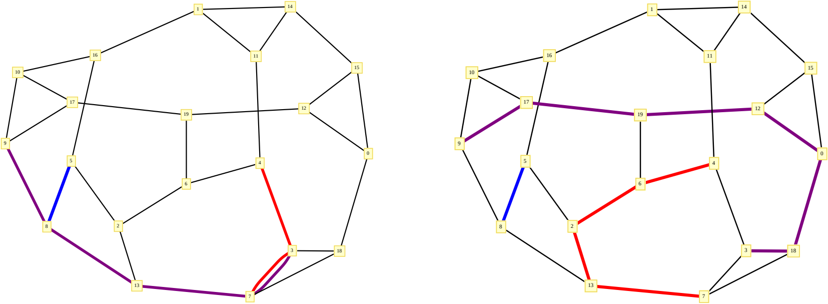
Our proposed solution:

$$E_{ij}(\bar{I}_{ij}) = \min_{\bar{I}_{ki}|constraint} \left\{ \sum_{k \in \partial i \setminus j} E_{ki}(\bar{I}_{ki}) \right\} + f(||\bar{I}_{ij}||)$$

Problem: exponential complexity $\approx 3^M$

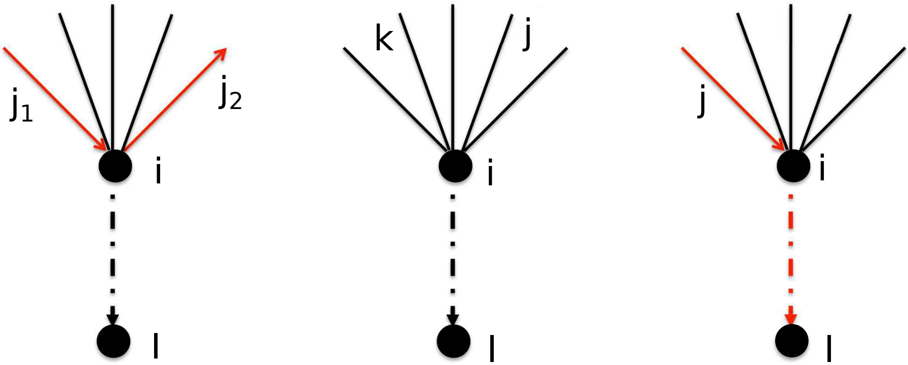
Node disjoint path (NDP)

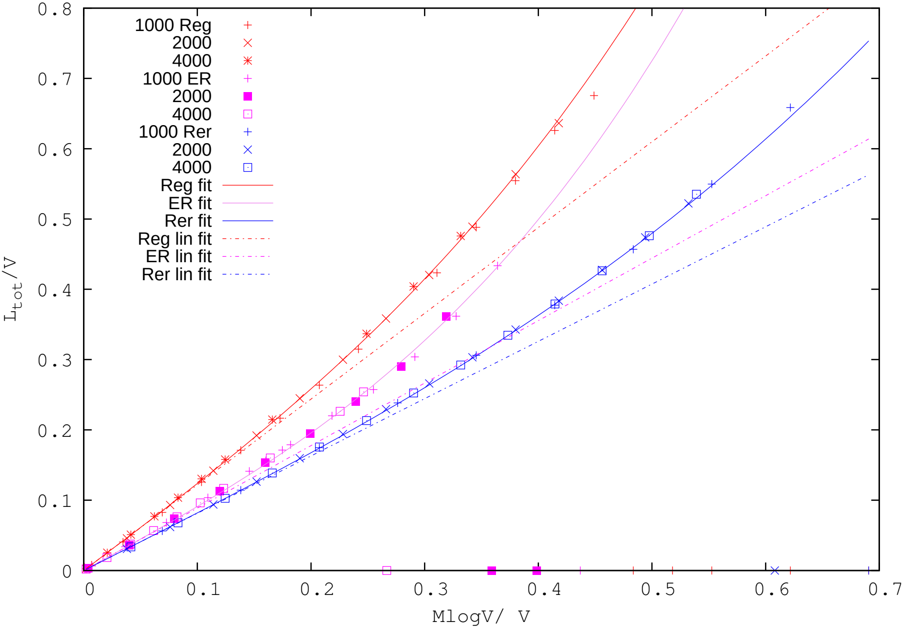
Decision problem: is there a NDP configuration that accommodates all the communications?



Shortest node-disjoint paths on random graphs, C De Bacco, S Franz, D Saad and C H Yeung. J. Stat. Mech. (2014) P07009

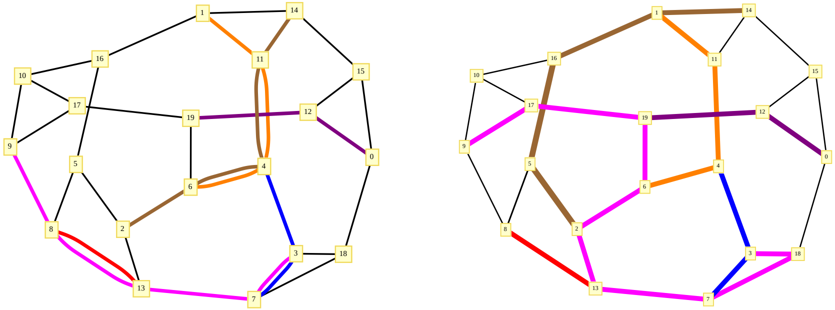
$$E_{ij}(\bar{I}_{ij}) = \min_{\bar{I}_{ki}|\textcolor{red}{constraint}} \left\{ \sum_{k \in \partial i \setminus j} E_{ki}(\bar{I}_{ki}) \right\} + f(||\bar{I}_{ij}||)$$




1000 Reg
2000
4000
1000 ER
2000
4000
1000 Rer
2000
4000
Reg fit
ER fit
Rer fit
Reg lin fit
ER lin fit
Rer lin fit
L_{tot}/V
MlogV/ V
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

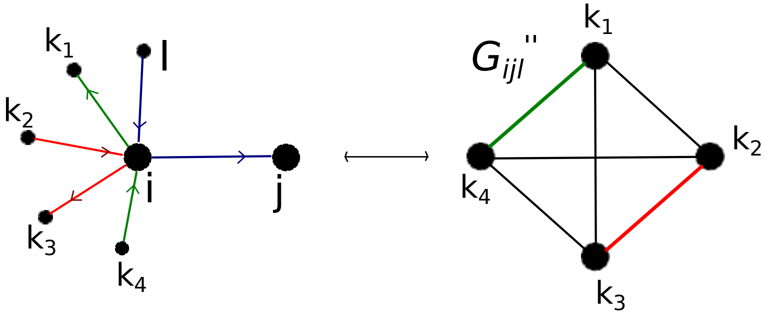
Edge disjoint path (EDP)

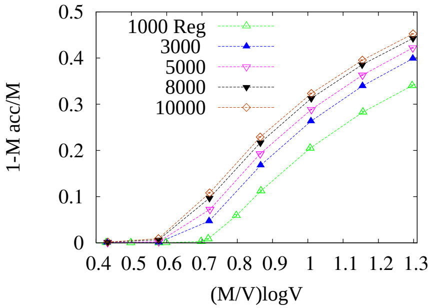
Optimization problem: what is the max number of communications accommodated?
(c.f. in the NDP the focus was on the decision problem)

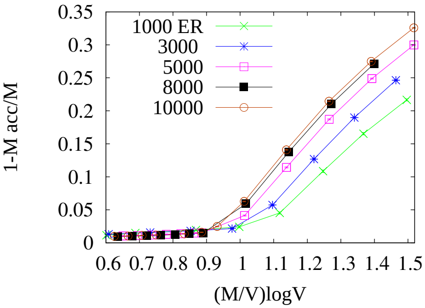


The edge-disjoint path problem on random graphs by message-passing, F. Altarelli, A. Braunstein, L. Dall'Asta, C De Bacco and S Franz. arXiv:1503.00540

Idea: Mapping edge-disjoint constraint on a star $\rightarrow$ combinatorial matching on a auxiliary complete graph




1000 Reg
3000
5000
8000
10000
1-M acc/M
(M/V)logV
0.5
0.4
0.3
0.2
0.1
0
0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3

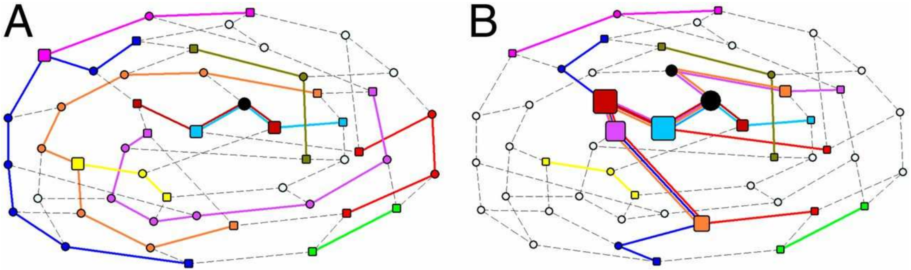

1000 ER
3000
5000
8000
10000
1-M acc/M
(M/V)logV
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5

Instance				MP			MP rein = 0.002			MSG (greedy)			ACO			LS		
Name	$\|V\|$	$\|E\|$	$\langle k \rangle$	$\langle M \rangle$	M_{min}	M_{max}	$\langle M \rangle$	M_{min}	M_{max}	$\langle M \rangle$	M_{min}	M_{max}	$\langle M \rangle$	M_{min}	M_{max}	$\langle M \rangle$	M_{min}	M_{max}
blrand10_1	500	1020	4.08	16.00	16	**16**	16.00	16	**16**	13.65	13	15	14.80	14	**16**	16.00	16	**16**
blrand25_1	500	1020	4.08	32.00	32	**32**	32.00	32	**32**	27.75	26	30	31.85	31	**32**	32.00	32	**32**
blrand40_1	500	1020	4.08	38.00	38	**38**	38.00	38	**38**	33.10	32	35	37.85	37	**38**	37.90	37	**38**
blrand10_2	500	1020	4.08	26.00	26	**26**	25.65	25	**26**	23.85	23	25	25.25	25	**26**	26.00	26	**26**
blrand25_2	500	1020	4.08	35.00	35	**35**	35.00	35	**35**	30.75	29	33	34.75	34	**35**	34.95	34	**35**
blrand40_2	500	1020	4.08	37.00	37	**37**	37.00	37	**37**	32.45	31	34	36.95	36	**37**	36.95	36	**37**

Instance				MP			MP rein = 0.002			MSG (greedy)			ACO			LS		
Name	$\|V\|$	$\|E\|$	$\langle k \rangle$	$\langle M \rangle$	M_{min}	M_{max}	$\langle M \rangle$	M_{min}	M_{max}	$\langle M \rangle$	M_{min}	M_{max}	$\langle M \rangle$	M_{min}	M_{max}	$\langle M \rangle$	M_{min}	M_{max}
mesh15_10_1	225	420	3.73	22.00	22	**22**	22.00	22	**22**	20.60	20	**22**	19.65	19	21	21.55	21	**22**
mesh15_25_1	225	420	3.73	36.00	36	**36**	35.10	35	**36**	28.30	27	30	27.70	26	29	32.00	31	33
mesh15_40_1	225	420	3.73	43.00	43	**43**	42.50	42	**43**	30.10	28	32	35.30	32	38	38.80	37	40
mesh15_10_2	225	420	3.73	-	-	-	19.89	19	**20**	19.75	19	**20**	17.50	17	19	19.45	19	**20**
mesh15_25_2	225	420	3.73	35.00	35	**35**	34.70	33	**35**	29.25	29	30	29.20	28	31	33.05	32	34
mesh15_40_2	225	420	3.73	42.00	42	**42**	41.35	41	**42**	29.80	29	32	34.00	33	36	37.60	36	39
mesh25_10_1	625	1200	3.84	-	-	-	47.25	46	**48**	40.70	40	42	32.85	29	36	41.00	39	43
mesh25_25_1	625	1200	3.84	-	-	-	68.30	67	**69**	48.40	47	51	45.00	42	49	55.55	54	59
mesh25_40_1	625	1200	3.84	-	-	-	88.74	88	**90**	54.35	53	58	57.70	53	61	69.30	67	72
mesh25_10_2	625	1200	3.84	-	-	-	44.33	43	**46**	40.05	38	42	30.10	28	33	37.90	36	40
mesh25_25_2	625	1200	3.84	-	-	-	67.22	65	**70**	48.90	47	52	45.60	44	48	54.70	52	59
mesh25_40_2	625	1200	3.84	-	-	-	88.55	87	**90**	54.05	51	57	57.75	54	61	68.85	66	71

Congestion optimization

Final step: remove disjointness constraint.



From the physics of interacting polymers to optimizing routes on the London Underground, CH Yeung, D Saad and K Y M Wong PNAS 2013 110 (34)

Open questions

- Can we adapt this formalism to study instead Nash equilibrium?
- Use a mean-field current $\bar{I}_{ij}$ that has to be determined self-consistently
- How to enforce combination of Nash and Kirchhoff as constraints?

Conclusion

Routing optimization

- Distributed algorithms to find the optimal flow configuration
- Path length and traffic are optimized
- Hard constraints NPD and EDP
- Question: how to generalize to soft constrains?

Thanks!